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The nonlinear dynamics of ship motions: a field overview and some recent developments

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This opening article of the issue is intended to provide a review of the field of nonlinear ship dynamics and to demonstrate the importance of the subject in the context of naval architecture. A list of relevant research topics that have received attention in recent years is included in the first section of the paper. This sets the scene for a brief discussion about the content and contribution of the articles that make up the Theme Issue. In order to facilitate understanding by non-specialists, we include an outline of the general concepts of nonlinear dynamics. Furthermore, we provide a detailed review of the evolution of ship research of the considered type since the 1970s, including a rather lengthy bibliography. We discuss the latest ideas about ship capsize and we include a special section summarizing the recent advances with regard to the mechanisms of ship capsize in astern seas. We close the article by offering some ideas about future directions for research.

Keywords: nonlinear; ship; bifurcation; chaos; capsize; broaching

1. Introduction

In the field of nonlinear ship dynamics we explore the onset and evolution of the unfamiliar, and often unsafe, dynamic responses beyond the linearity regime, which are not amenable to the conventional techniques of seakeeping theory. The presence of nonlinearity in the relation between excitation and response in a dynamical system creates the prospect of having multiple solutions for certain values of the system's parameters. This could further lead to a plethora of very complicated phenomena even when the type of nonlinearity is very simple; such as a quadratic or cubic term in the stiffness component of an otherwise ordinary linear driven oscillator (Thompson & Stewart 1986). Unexpected and violent transient motions leading to system escape, irregular behaviour under regular excitation, subtle boundaries between the domains of coexisting responses, and sensitive dependence on initial conditions are some well-known manifestations of nonlinear behaviour.

The above ideas are not yet fully domesticated in naval architecture, a field that is basically dominated by 'linear' concepts. However, nonlinear phenomena have been found to underlie ship dynamics in several instances, much as they do for the dynamics of other engineering systems (Thompson & Bishop 1994; Moon 1999). Scientific curiosity apart, their particular engineering significance stems from their frequent connection with safety-critical behaviour. For example, the roll stability of a ship (the term stability used here in a liberal sense) is considerably reduced once it is exposed to near resonant waves with steepness higher than some critical level. The reason is rapid loss of integrity of the domain of bounded (and thus safe) roll, due to a purely nonlinear mechanism. In other cases, variation of certain system parameters could suddenly render viable large motions with unusual characteristics. These varied parameters could either be representatives of the environment in which a ship operates, like the wave height and frequency, wind speed, etc., or they could reflect the ship's own operational and design characteristics, like the natural frequency and the damping in a direction of motion, the desired ship course, etc.

By using the conceptual and computational framework of nonlinear dynamics, ship scientists around the world have recently produced some wonderful new insights into phenomena of ship motion instability. An ensemble of 'geometry-led' numerical and analytical techniques has been the basis of these investigations, applied in conjunction with mathematical models of various degrees of sophistication. It is very encouraging that descriptions of phenomena experienced in the ocean by mariners often coincide closely with the theoretically predicted patterns. Research topics that have received particular attention recently are listed here.

(1) Ship capsize in regular 'beam' seas.

(2) Development of transient capsize criteria based on Melnikov's method.

(3) Criteria of ship capsize in random waves.

(4) Large-amplitude rolling and capsize in longitudinal waves.

(5) Single-degree and coupled surge dynamics; especially the occurrence of asymmetric surging and surf-riding.

(6) Broaching and its connection with capsize.

(7) Coupled rolling with liquid motion in an internal compartment.

(8) The behaviour of high-speed planing craft; especially the heave-pitch oscillations known as the porpoising condition.

(9) Oscillations of moored, anchored or towed ships under the effects of currents (mainly for yaw motion) or waves (mainly for surge).

(10) Path control of marine vehicles and directional instability in strong wind.

Some of these achievements will be reviewed below. Awareness of the fund of knowledge created by these investigations—combined with some familiarity, at a user level, with the techniques of nonlinear dynamics—will open new horizons for practising naval architects in their quest to maximize ship safety.

Increased computer power nowadays allows us to carry out investigations on the basis of mathematical models featuring a considerable degree of detail. Indeed, one of the greatest challenges for the future will be interfacing, and, ultimately, integration, with the advances currently taking place in the area of numerical ship hydrodynamics (Beck 1996; Huang & Sclavounos 1998). This will allow an explicit account of the flow around ships' hulls and will supply the device for investigating the effect of the shape of the hull and the appendages on the nonlinear phenomena. Such an optimistic view is somehow counterbalanced, however, by the fact that the burden of identifying areas of critical behaviour becomes very considerable when the dynamical system is multi-dimensional (let alone when it is infinite dimensional, as is the true nature of the problem of a solid body's motion in a fluid). The derivation of reduced-order approximations of the original system, either through some rational dimension reduction procedure, or, merely, and usually more effectively, through insightful thinking, is very topical and will probably remain so because simple models are indispensable for developing an understanding about complex processes. To quote Wehausen (1979) in his tribute to Weinblum: 'a direct attack upon a too complicated problem may be in danger of not uncovering the underlying principles'. Much of the current interest in nonlinear ship dynamics was stirred by the success of the approach in unravelling, in a systematic way, the principles governing unsafe behaviour on the basis of simple generic models.

2. Content of the present Theme Issue

With the present Theme Issue we have attempted to present the various facets of ongoing research in the field. Contributions from the following areas have been included.

(1) Investigations on instability phenomena based on simplified models.

(2) Criteria of nonlinear rolling and capsize in a random-wave environment.

(3) Studies of nonlinear behaviour with detailed mathematical models.

(4) Observation and physical verification of complex behaviour.

The first paper is from Jiang *et al.*, and it is the latest addition to an impressive series of papers produced by 'the Michigan group' over the years on this subject. In their present work they attempt to extend the Melnikov analysis applied for the single-roll capsize problem in a random sea in order to include the so-called 'memory effect'. In many studies of nonlinear dynamics it has been quite common to assume that the hydrodynamic force is only a function of the instantaneous ship motion. In a strict sense this is not accurate, because, generally, the whole history of motion plays a role due to wave radiation from the ship's oscillation and/or viscous effects. Taking this feature into account would require inclusion of convolution integrals leading to an integrodifferential form of motion equations and infinite-dimensional state space. These authors have used the concept of phase space flux in their study, and they approximated the memory effect through an auxiliary system model, whose output represents an excitation to the original non-memory one. They have carried out the Melnikov analysis of the enlarged system and they have shown some examples of how the memory term could influence the critical wave height for capsize. Some comparisons with non-memory models using constant characteristic frequencies are also presented.

The work of Murashige *et al.* at the University of Tokyo deals with the problem of ship motion when some quantity of water exists in an internal compartment. This problem has received considerable attention recently because it can give rise to robust chaos. These authors have carried out both numerical and experimental studies, and, despite a simple lumped-mass approach to the modelling of internal water movement, the agreement between some of their findings (figures 2 and 13 of their paper) is encouraging. They have used two different mathematical models, the simpler of which has the form of a coupled Duffing's equation with a bistable restoring term and a nonlinear inertial matrix. An interesting account of the occurring bifurcations is given. The authors also speculate that the cause of the chaotic responses is a chain of codimension-two bifurcations. Analyses of experimental data are presented in the paper, including data-smoothing techniques, phase-space reconstruction with delay time parameters, calculation of Lyapunov exponents, and derivation of attractor dimension.

The paper of Spyrou of the National Technical University of Athens is an analysis of the coupled roll and surge dynamics. In steep following waves, the nonlinear features of the pendulum-like surge dynamics may become prevalent, resulting in a motion pattern in which the ship is spending more time near the crests of the waves than near the troughs. However, as restoring is lowest around a crest, such a prolonged stay at the crest can considerably influence the propensity for capsize. Inspired by this idea, the author has developed a new equation of roll motion in a following sea, which takes into account the surge dynamics. Another useful facet of this paper is the demonstration of the layout of the stability transition lines of the coupled system, which are contrasted with those of the ordinary Mathieu-type system. In particular, the evolution of the capsize region in conjunction with the emergence of surf-riding behaviour due to the occurrence of a homoclinic connection is very interesting. It seems that these phenomena tend to raise the probability of a quick capsize, and, therefore, their omission could result in underestimation of the safety margin of a ship.

The paper of Kreuzer & Wendt informs us about ongoing work at the Technical University of Hamburg–Harburg, where the main focus is the study of nonlinear phenomena linked with ship capsize. The authors are using a full six-degrees-of-freedom mathematical model with frequency-dependent hydrodynamic coefficients. A standard two-dimensional singularity method is used in the calculation of the hydrodynamic forces, while the inclusion of 'memory' has led to a state-space representation with 164 degrees of freedom. Their investigations are carried out initially for a fixed rudder, but later they have used a proportional integral differential (PID) autopilot in order to control the rudder's movement. They have presented animations of realistic ship behaviour in waves during capsize in (stern) quartering waves. Some subharmonic and chaotic responses are identified in extreme waves.

The paper of Oh, Nayfeh & Mook from Virginia Tech describes the main findings of a combined theoretical and experimental investigation on indirectly excited rolling in head or following seas. Two specific phenomena are analysed in detail: the first is a parametric resonance case in roll caused by the pitch and heave motions of a ship in waves. The equations describing heave and pitch are considered decoupled; however, the roll equation presents nonlinear couplings with both of these motions. The main safety-related finding here is the discovery of a subcritical pitchfork bifurcation and it is very notable that its existence was confirmed during experiments with a model ship. Another case analysed in this paper is autoparametric resonance for coupled pitch and roll. Again dangerous subcritical-type instability phenomena were found. This paper shows that it is certainly possible to correlate theoretical findings with the results of model experiments, despite the complex character of the behaviour and the many parameters that are involved in such experiments.

Umeda & Hamamoto of Osaka University have collected some very interesting visual images of ship behaviour in connection with four different capsize modes in high-stern-quartering seas. These images were obtained during experiments based on radio-controlled models. The experiments were carried out at the Marine Dynamics Basin of the National Research Institute of Fisheries Engineering of Japan. The authors have reproduced capsize due to broaching and due to parametric instability. Also, they have shown capsize according to two less frequently discussed modes: capsize after bow diving; and capsize due to a kind of pure loss with drift motion involved.

The final contribution featuring model experiments comes from Ikeda & Katayama of the University of Osaka Prefecture and is focused on ship behaviour at high speed. These authors have studied the onset of the so-called porpoising oscillations (self-sustained pitch and heave) for a marine craft with spray rails operating up to a Froude number as high as 6.0. On the basis of a one-quarter-scale model they measured the forces with captive tests for various running attitudes and speeds, and they found nonlinearity for heave and pitch restoring at high speed, especially for the cross-coupling terms. This information was introduced into a mathematical model in order to simulate the initial occurrence and further build-up of porpoising. The derived predictions of porpoising characteristics were compared with direct experimental measurements and found to be in good agreement.

The last three contributions are concerned with the characteristics of nonlinear rolling in a random seaway. Roberts & Vasta of the University of Sussex have applied an averaging procedure, showing that, under certain conditions, the energy envelope of the roll response can be modelled as a continuous Markov process. Their approach could be useful for deriving probability density functions of roll response when the time to capsize is very long. Another potential use of this work is in the derivation of damping and excitation data from the so-called drift and diffusion coefficients calculated on the basis of actual ship roll-response information.

The papers of Senjanović and Falzarano with their associates are more mainstream nonlinear-dynamics investigations of ship capsize under stochastic wave excitation. These works are highly complementary, as they seem to approach the same problem from a different perspective. Senjanović, Ciprić & Parunov from the University of Zagreb followed a repetitive simulation procedure in order to produce a chart giving the probability of capsize (for a certain wave spectrum) in terms of ship-motion direction and speed. As usual, they have derived the (irregular) wave forcing by a sum of a finite number of harmonic wave components with random phases. As the detailed form of the righting-arm (GZ)-curve is taken into account, this chart could be used also for linking quantitatively the shape of the (GZ)-curve with the probability of capsize.

Instead of performing many simulations, Vishnubhotla with Falzarano (University of New Orleans) and Vakakis (University of Illinois at Urbana) attempted to derive approximate analytical expressions for the inset and outset of the hill-top saddle under pseudo-random-wave excitation. They used techniques from the theory of rapidly varying excitations and they applied for a cubic-type restoring function whose Hamiltonian manifold solution is well known. The analytical approach could have some problems for more realistic higher-order restoring polynomials; however,

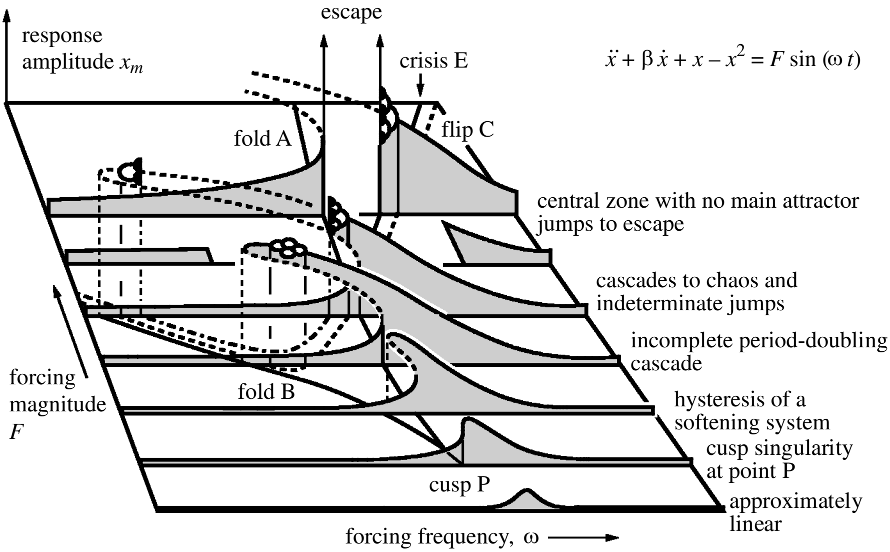

Figure 1. Evolution of steady-state responses for increasing excitation based on a roll equation allowing only one-sided escape (Thompson 1996).

the authors believe that the numerical route could be applied without problems in these cases.

3. Introductory aspects of nonlinear dynamics

A simple, harmonically forced oscillator with a quadratic 'softening' nonlinearity in restoring has been used as a generic model for studying various physical phenomena, one being one-sided ship capsize in 'beam' waves (Thompson 1997). The evolution of the responses as the forcing is stepped up amply demonstrates several characteristic features of nonlinear systems (figure 1). Most familiar are the 'skewed' (to the left for softening nonlinearity) frequency-response curve and the gradual appearance of higher harmonics 'distorting' the ordinary harmonic motion. At a certain level of forcing, the effect of nonlinearity becomes so drastic on the upper limb that the original response turns unstable with a stable subharmonic emerging. Qualitative changes in the character of behaviour are manifestations of *bifurcations*, which are very common phenomena in nonlinear systems. They can be realized according to a small number of generic patterns, which may involve smooth or abrupt transition towards another state.

The discovery and in-depth study of bifurcations are key tasks in the exploration of ship dynamics. The robustness of the bifurcations in variations of the mathematical model allows us to draw certain conclusions about the behaviour of the system without necessarily having a complete mathematical model (almost universally the case when studying extreme ship motions in waves). This property could be exploited during free-running ship model tests in large waves, where making direct comparisons between theory and experiment is non-trivial, as the number of unknowns is very large and some of the parameters are practically non-controllable. The minimum number of system ('control') parameters that render a bifurcation robust

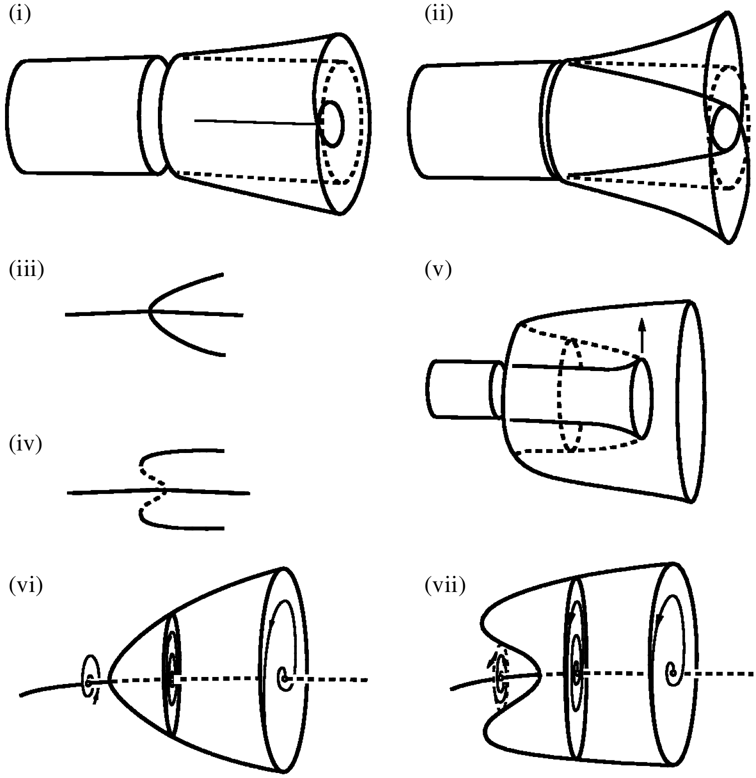

Figure 2. Codimension-one local bifurcations: (i) flip, (ii) pitchfork. Both of these may appear as: (iii) supercritical, or (iv) subcritical. Other possible bifurcations are: (v) the folds, (vi) supercritical Hopf and (vii) subcritical Hopf.

('structurally stable') determines the *codimension* of this bifurcation. Bifurcation phenomena are generally classified as local or global. Local bifurcation phenomena correspond to the creation, disappearance or change in stability of a steady-state solution. Local, codimension-1 bifurcations are the fold (also known as the saddle-node bifurcation, turning point or limit point), the pitchfork, the flip (period doubling), and the Hopf bifurcation (figure 2). All these have confirmed relevance to ship motions. Of particular interest for engineering systems are those bifurcations that do not create a stable state in the vicinity past the bifurcation point. At a fold, no solution exists at all, and a jump towards some distant state or even towards infinity becomes inevitable. A similarly dangerous jump will arise at a pitchfork, flip or Hopf bifurcation if these are of a *subcritical* type. A classification of generic bifurcations for dissipative dynamical systems is given in Thompson *et al.* (1994).

However, the study of bifurcations in a steady-state sense does not offer a complete account of nonlinear behaviour. Unsafe transitions are, by their very nature, extreme unsteady phenomena. Also, in the continually changing conditions of the ocean, a ship will only rarely operate in a steady condition. The level of excitation required for system escape during a transient is lower than that of the steady state, and, thus, safety-margin predictions based on steady states could overestimate the true safety margin. The assessment of critical behaviour requires, therefore, some understanding of the principles governing transient motions. Generally, such an understanding can be acquired more easily if a geometric approach is adopted in which the motions' characteristics are studied in the so-called phase (or state) space, or through its two-dimensional projections. The phase space is basically an enlarged physical space. Its dimension is determined by the number of initial conditions required in order to obtain, from the motion equations, a unique solution in time. In the case of undriven rolling, for example, it suffices to know the initial roll angle (i.e. the value at $t = 0$) and the initial roll velocity, in order to determine the solution at a later time instant; so the state-space dimension is, in this case, 2. The continuous time evolution of a solution produces a *trajectory* (also called an *orbit*). Such trajectories cannot cross in phase space; otherwise, at the intersection point there would exist non-unique evolution. Stationary steady solutions are represented in state space by points, while periodic solutions appear as closed curves. Stable solutions are called *attractors* (or *sinks*) because the trajectories starting from nearby points tend to end on these. The unstable solutions are called *repellers* (or *sources*). Quite often we may have attraction in certain directions and repulsion in the others. This is the case of *saddle*. The evolution of initial conditions produces a flow in phase space. When the steady response contains incommensurate frequencies we have a *quasi-periodic* attractor or repeller ('leaving' on a torus). In some cases the response is characterized by broad-band noise and extreme sensitivity to initial conditions, despite the fact that the system is forced regularly. This is the case of a *chaotic* response. Typical routes to chaos are through a cascade of period-doubling bifurcations, as exhibited in figure 1 (the ratio of the distance between successive bifurcation frequencies converges to the universal constant 4.669, first discovered by Feigenbaum); through quasi-periodicity and through intermittency (Bergé *et al*. 1984). The first two routes have been discussed in the context of ship motions (Virgin 1987; Rainey & Thompson 1991; Spyrou 1996*b*; Murashige & Aihara 1998*a*).

A key concept required for the study of transients is the *basin* (*domain*) *of attraction*, representing the set of initial conditions that lead to a certain attractor for fixed control parameters' settings. Conversely, the basin of attraction may also be defined in the control space for fixed initial conditions. For a simple two-dimensional system the boundary of the basin of attraction is determined by those special orbits that approach (in infinite time) nearby saddle-type states (figure 3). Together with their 'twin' orbits, which leave these saddles, they are usually called *invariant manifolds*, or, more correctly, *inset* (if entering a saddle) and *outset* (if leaving a saddle). These manifolds can sometimes become entangled in a very intricate way, producing boundaries that are not sharp. The critical event leading to such a development is the tangency of a pair of manifolds originating from the same ('homoclinic') or from different ('heteroclinic') saddles. This is a *global bifurcation* phenomenon creating a major phase-space rearrangement (figure 3). Such phenomena are very critical for the system's integrity because they are associated with rapid loss from within the area of the safe basin (figure 4). They also generate transient chaos: orbits started at certain initial conditions wander in an apparently random manner, albeit that they settle to a repeating pattern in the long term. Further stepping-up of a critical control parameter, however, such as the forcing amplitude, is likely to create chaos in the more familiar steady-state sense too.

Heteroclinic/homoclinic tangencies have been linked to the process of ship capsize in resonant beam seas (Thompson & Soliman 1990). Their occurrence can be pre-

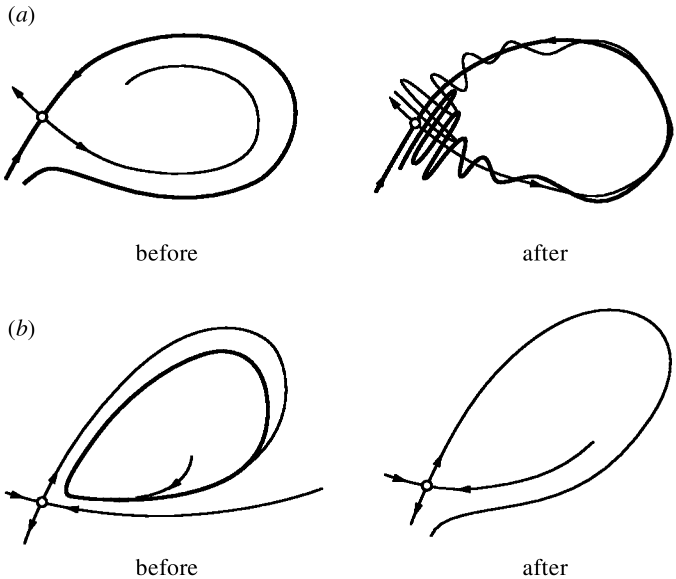

Figure 3. Global bifurcations with known relevance for ship dynamics. (*a*) Homoclinic tangency in a map; (*b*) homoclinic saddle connection.

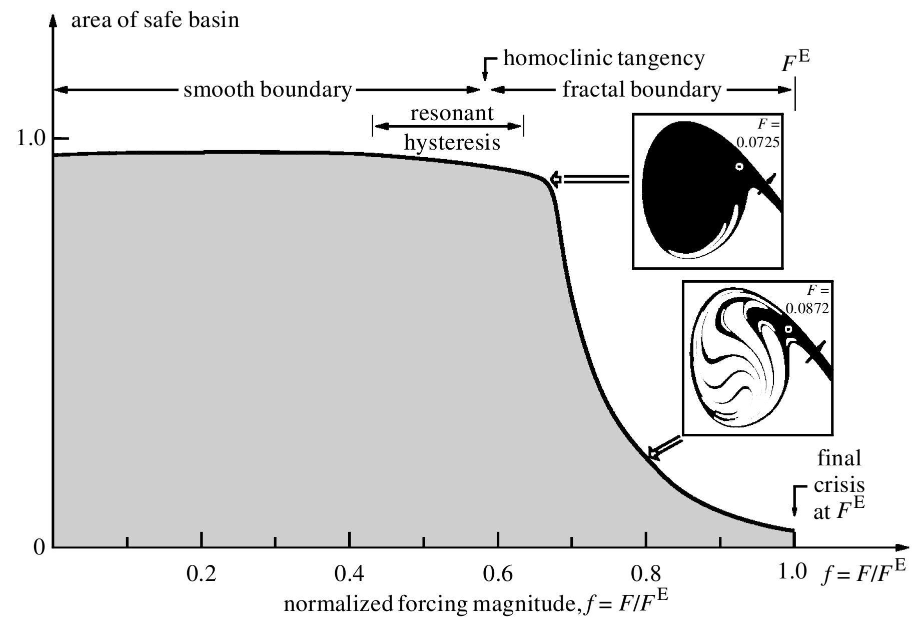

Figure 4. Loss of engineering integrity due to erosion of the safe basin of attraction (Thompson & Soliman 1990).

dicted either numerically or, in an approximate sense, analytically. The analytical route has been very popular in the recent past, and is based on the so-called Melnikov method of analytical mechanics, which, in a strict sense, is valid only for infinitesimal damping (e.g. Guckenheimer & Holmes 1983). The main task in this method

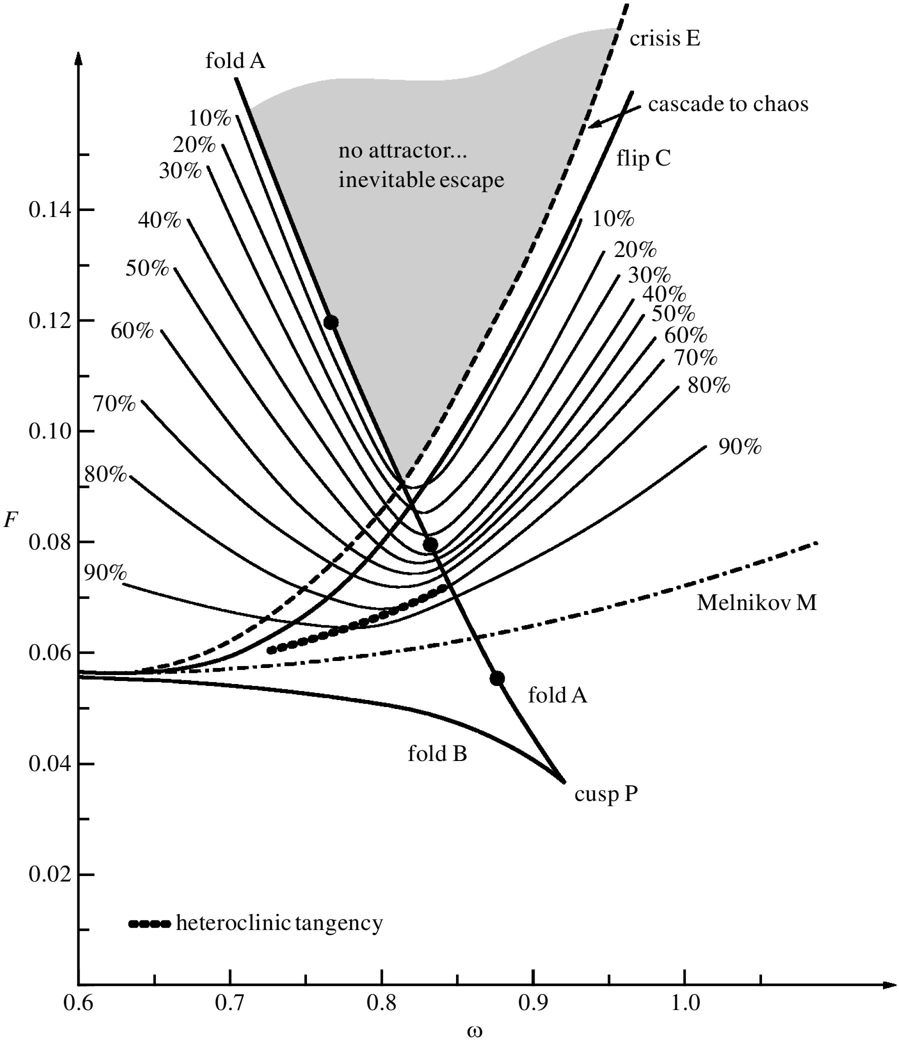

Figure 5. Global integrity contours for the 'one-sided-escape' equation. Remaining safe basin area shown as a percentage, $\beta = 0.1$ (Thompson 1996).

is to express the distance between the critical pair of manifolds as a function of the ship and environmental parameters. Then, by setting this distance to zero, we can determine the critical combination of these parameters. This condition is equivalent, with a balancing of the energy coming in through forcing with the energy dissipated through damping, for the remotest bounded orbit (for a Hamiltonian system, this is the homoclinic one). The bifurcation diagram of the simple oscillator of figure 1 including the Melnikov curve as well as the numerical prediction of the homoclinic connection is shown in figure 5.

Another type of global bifurcation that has manifested itself in ship dynamics is the so-called *homoclinic saddle connection*, which is a dangerous 'blue-sky' event. A saddle 'approaches' an existing, stable, periodic orbit until there is contact (equivalently, an inset–outset pair of the saddle touches the periodic orbit tangentially).

The outcome is the sudden disappearance of the periodic orbit from phase space (figure 3). Such an event has been identified to be behind the so-called *surf-riding* condition, and it is the main cause of *broaching* (Spyrou 1996*a*).

It is quite straightforward to identify basins of attraction and extract the phase-space organization through discretization of the phase space and direct integration from the nodes of a dense grid of initial conditions. A more efficient, computationally, version of this method is the cell-to-cell mapping technique, which ensures that points whose evolution is known are not examined repetitively (Hsu 1987). These methods are generally very successful for two-dimensional Poincaré sections (a Poincaré section is basically a section through phase space transverse to the flow that allows the study of the dynamics on a plane), but their efficiency falls quickly as the phase-space dimension is increased. For example, for a typical 12-dimensional phase space required for the general study of ship dynamics in all six degrees of freedom and a modest grid density of 300 nodes in each direction, simulations for 300^{12} points would be needed! More intelligent searching techniques are therefore required. The analysis of multi-degree-of-freedom systems is a new field still under development.

4. Historical note on nonlinear ship dynamics

The forerunners of the modern approaches are traced back to the 1970s: Zeeman's (1977) reformulation of some classical results of roll stability from the perspective of catastrophe theory bears a strong resemblance to many of the popular geometrical approaches used today (see also Poston & Stewart 1978). In the 1970s, the study of large-amplitude rolling in beam waves became topical, perhaps due to the SAFESHIP project funded by the UK government. Analytical approximations of the steady-state rolling response with higher harmonics were derived by applying the asymptotic, the harmonic balance, and other perturbation-based methods (Odabashi 1973; Wellicome 1975; Wright & Marshfield 1980). In these studies, roll was uncoupled and the waves were considered long compared with the ship's beam, resulting in a mathematical model that is basically a classical Duffing-type forced oscillator with a single potential well. Actually, these were not the first explorations of the nonlinear roll equation; for example, Baumann (1955) nicely demonstrated the dependence of the shape of the frequency-response curve on the form of realistic (GZ)-functions.

Marshfield (1978) published the first results of the well-known 'Admiralty model tests', providing concrete evidence about the nonlinear character of the frequency-response curve and the existence of bistability in a certain range around resonance (see also Marshfield 1987). Records of subharmonic responses were also presented, verifying that the roll motion may undergo a period-doubling bifurcation. Analytical investigations on subharmonic rolling were performed later by the Trieste group (Cardo *et al*. 1981). Marshfield has also made the important observation that the existence of bias towards and away from the wave source produces different results in terms of capsize; and that the unbiased model shows a tendency to capsize towards the wave source. Sway coupling may be necessary for explaining these tendencies, but roll is quite often studied uncoupled. Some simplifying assumptions about the roll centre are necessary in order to justify this model (Hutchison 1990). It is quite common to describe the roll motion on the basis of the relative roll angle as measured from the local wave slope. Of course, the absolute roll angle, measured from a horizontal line, may also be used (see Blagoveshchensky 1962; Francescutto & Contento

1998). Analytical studies of steady rolling in beam seas continue to be carried out up to the present day (Senjanović 1994; Peyton Jones & Cancaya 1997).

Odabashi seemed to be the first person to make the connection with the classical theory of stability, recognizing that the domain of attraction, rather than the steady-state, holds the key to the assessment of ship safety against capsize. He proposed the derivation of criteria for ensuring the boundedness of roll motion through the analytical construction of the so-called Lyapunov functions (Kuo & Odabashi 1975; Odabashi 1977, 1978, 1982). These Lyapunov functions were hotly debated in the 1970s and 1980s in the naval architectural community, which was perhaps still unprepared for adopting such highly mathematical techniques.

In the United States, Nayfeh *et al.* (1973) published a paper about nonlinearly coupled pitch and roll using the method of multiple scales. In head seas, directly excited pitch can create growth of roll, which will influence back pitch, and so forth. A critical development is the saturation of the pitch response and the transfer of all the energy that enters the pitch mode into roll. As stated in their introduction, the authors had been inspired by Froude's (1863) observation, published in his discussion of Scott Russel's paper at the Institution of Naval Architects, that ships present undesirable roll characteristics when the natural frequency of pitch is twice that of roll. Sethna & Bajaj (1978) predicted further amplitude-modulated motions in the averaged equations of coupled roll and pitch. Nayfeh has come back to this problem several times since then (see, for example, Nayfeh 1988; Oh *et al.* 1992). It is interesting to mention that Paulling had approached this problem earlier from a different angle, investigating whether a ship could maintain the 'upright' equilibrium position in calm sea when roll is coupled with a prescribed heave or pitch motion (Paulling & Rosenberg 1959; Paulling 1961).

At that time, it was already known from the works of Grim (1952, 1954) and Kerwin (1955) that roll instability could be generated by the restoring becoming time dependent when a ship encounters a long regular wave travelling in the same direction. Fundamentally, this problem, as well as the problem studied by Paulling, can be reduced to a study of an equation of motion with a harmonically varying stiffness term ('Mathieu' type). These pioneering studies have inspired significant research subsequently (see, for example, Blocki 1980; Sanchez & Nayfeh 1990; Kan 1992). Liaw *et al.* (1992) found chaos in heave-excited roll of a barge in head seas.

The interest in nonlinear ship dynamics started to surge in the 1970s; but at that time the computers were still at an embryonic stage and it was perhaps inevitable that prominence would be given to analytical methods. Unfortunately, these methods are not only extremely laborious but they also require that the nonlinearities be weak, thus eliminating much of the most dynamically interesting part of the behaviour. For the ship-capsize problem in particular, while the nonlinearity of damping can be considered as mild, the nonlinearity of restoring is strong, and it is inconsistent to examine capsize without considering a fully nonlinear restoring curve. A different viewpoint would thus be needed if an effective solution were to be achieved.

The vast increase in computer power realized in the 1980s and 1990s brought a wide range of new techniques of numerical analysis of nonlinear dynamical systems into the limelight. These went well beyond the ordinary simulation based on direct integration of the equation of motion. Numerical algorithms were developed for carrying out continuation of steady states; for directly locating bifurcation points; for identifying the stable and unstable manifolds either directly or through a vast number of automated simulations that unveil the basin of attraction (for a comprehensive overview see, for example, Foale & Thompson (1991)). Techniques for the characterization of the responses and also for the extraction of possible differentiable dynamics from experimental data series have also been developed.

The technique of steady-state continuation (also known as 'path following') was applied very quickly to the neighbouring field of flight dynamics (Mehra & Carroll 1980).† However, another decade would pass before the technique was used in the study of multi-degree ship motions. Continuation is the technique employed in order to determine how a steady motion changes quantitatively as well as qualitatively when some control parameter is varied. Numerical codes of this kind, tracing stationary and periodic states and passing successfully over bifurcation points, have been written by a number of people (Keller 1977; Doedel 1981; Kubicek & Marek 1983; Holodniok & Kubicek 1984; Rheinboldt 1986; Seydel 1988; and others). Continuation based on Kubicek's method, coupled with simultaneous stability analysis, was used for investigating the directional stability of ships with a surge, sway, yaw and roll mathematical model (Spyrou 1990). Diagrams of steering for multi-degree-of-freedom ship manoeuvring, derived for operation in calm conditions and also in uniform wind, showed the existence of fold and Hopf bifurcations. At about the same time, continuation was employed for the study of some aspects of the behaviour of a ship slowly turning in waves (Falzarano *et al*. 1990).

Extensive studies of the bifurcating behaviour of marine vehicles, on the basis of simulation or by extracting simple generic models, were carried out at the University of Michigan in the late 1980s. These studies were focused on the behaviour of moored tankers (single point initially, and, more recently, multi-line), and they pointed out the existence of pitchfork, Hopf and period-doubling bifurcations and also the possibility of chaos (Papoulias & Bernitsas 1988; Chung & Bernitsas 1992). A problem of a similar nature (the behaviour of moored or anchored tankers) was studied in Germany (Jiang *et al*. 1987; Sharma *et al*. 1988; Schellin *et al*. 1990). Researchers there carried out stability analyses on the basis of a quite-advanced surge–sway–yaw mathematical model that included the so-called memory effect discussed above. Aghamohammadi & Thompson (1990) studied experimentally 'fish-tailing' instabilities of a tanker at a single point mooring. Gottlieb & Yim (1993) predicted instability and chaos in a multi-point mooring system modelled with a simple surge equation having nonlinear restoring and a coupled wave-structure exciting force. Choi & Lou (1993) studied the low drift motion of a tanker induced by the nonlinearity of the mooring line. Jiang (1997) found a Hopf bifurcation creating yaw oscillations in a tug-tanker tow, depending on the towline length. Control studies on the bifurcations of marine vehicles were carried out by Papoulias (1991) and Papoulias & Oral (1995).

In all these years the study of nonlinear rolling in beam seas continued to be topical. Nayfeh & Khdeir (1986a, b) and Papanikolaou & Zaraphonitis (1987) presented studies of large-amplitude rolling based on a combination of analytical perturbation-based techniques and digital–analogue simulations. Virgin (1987) concentrated on the onset of chaotic roll oscillations occurring through a period-doubling cascade, which he observed on the basis of Poincaré maps. Thompson and co-workers (Thompson & Soliman 1990; Rainey & Thompson 1991) offered a new perspective on the ship-capsize problem by considering the capsize process as dynamically equivalent

† Recent developments in aircraft dynamics are summarized in a Theme Issue of *Philosophical Transactions of the Royal Society of London* (see Thompson & MacMillen 1998).

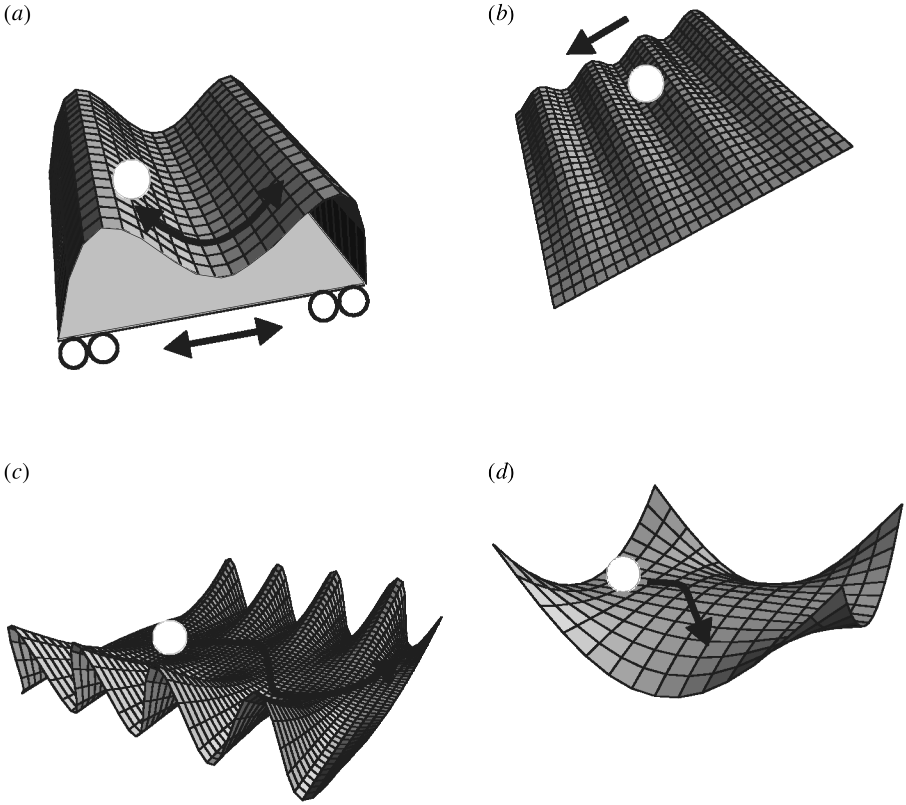

Figure 6. Simple analogues for ship dynamics in waves. (*a*) Beam sea rolling; (*b*) surging and surf-riding; (*c*) parametric; (*d*) pure loss.

to the escape of a ball rolling in a potential well, which is an intrinsically transient phenomenon (figure 6). One of the first major contributions was the proposal of a diagram for the practical assessment of a hull's capsizability, which became known as the *transient capsize diagram* (figure 7). Extensive cell-mapping and continuation studies were performed for a roll equation with direct or parametric forcing (see, for example, Thompson *et al*. 1992). The efforts to translate the information about fractal basin boundaries into design criteria against transient capsize were very notable. A simple design formula, derived from the displacement magnification of the linear resonance and validated by Melnikov theory and simulation, was put forward. Another significant finding was that the maximum ('sustainable') wave slope at which capsize is still resisted is very much reduced by the existence of even a small amount of bias (figure 8). Thus, a stability assessment based on a symmetric ship is bound to give a dangerous non-conservative capsize threshold (for a review of this research see Thompson (1997)). A recent study of the coupled roll and heave revealed a capsize-suppression mechanism introduced by the heave coupling (Thompson & de Souza 1996).

In the late 1980s/early 1990s, capsize due to transient rolling had become a 'hot' topic. In Japan, Kan (1992) followed a similar line of research to that of Thompson, with the important addition, however, of extensive model experiments in the

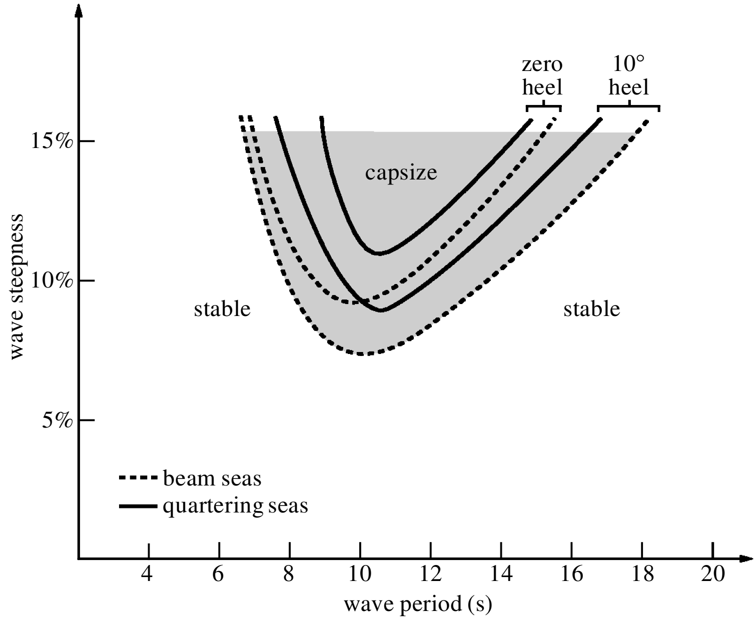

Figure 7. Transient capsize diagram (Rainey & Thompson 1991).

large open square tank of the ship dynamics division of the Ship Research Institute of Tokyo. In the USA, Nayfeh & Sanchez (1990) presented numerical safe basins for the roll motion in beams and in longitudinal waves (Sanchez & Nayfeh 1990). Another significant step forward was the use of Melnikov analysis for predicting capsizal wave slopes in beam regular waves (Falzarano *et al.* 1992). Use of realistic restoring representations, like a fifth- or higher-order polynomial, is, however, problematic when the analytical route is followed (Scolan 1998). Bikdash *et al.* (1994) examined the equivalence between quadratic and cubic damping nonlinearities from a Melnikov perspective. Hsieh *et al.* (1994) adapted the Melnikov approach for a random excitation using the concept of Wiggins (1991) for phase flux transport out of the safe basin. This approach was extended for a biased vessel by Jiang *et al.* (1996). The interfacing of nonlinear dynamics and stochastic excitation has also been attempted by others (see, for example, Francescutto 1990).

Liquids inside tanks or a quantity of water shipped on the deck can incur a significant effect on the roll dynamics. This problem is a complex one, especially when impacting loads due to sloshing can take place, and it has been tackled using methods of varying sophistication (for a review of some earlier ideas see Caglayan & Storch (1981)). One of the first studies was by Dillingham (1981), where the solution of the deck-flow problem was coupled with the body's roll motion. Recently, Armenio & Francescutto (1996) followed a similar approach, but they tackled the internal fluid-motion problem by solving the Reynolds averaged Navier–Stokes equations (RANSE). An approach with a different focus was that of Murashige & Aihara (1998*a*, *b*), who substituted the internal liquid with a lumped mass placed at the instantaneous centre of gravity of the liquid, which was further assumed always to have a flat surface. This simplification did not take into account sloshing or hydraulic jump effects, which certainly affect the dynamics; but, on the other hand,

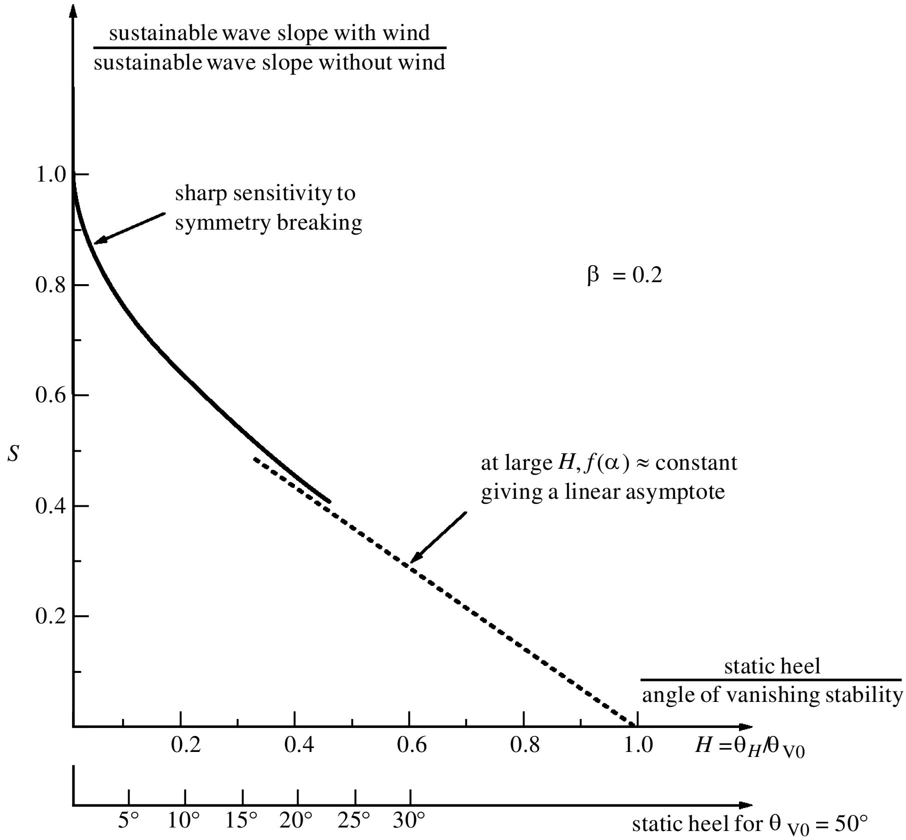

Figure 8. Sensitivity of capsize to bias (Thompson 1997).

this approach revealed a rich content in terms of nonlinear behaviour and also robust chaos. The authors showed that some nonlinear features could easily be reproduced experimentally in the laboratory.

Despite the fact that most of the nonlinear-dynamics research has been inspired by the capsize problem, it is well known that nonlinear behaviour may also originate in other motion directions. For example, many ships are *directionally unstable*, and their steering diagram is S-shaped, tending to turn to port or to starboard depending on the initial condition, even with the rudder undeflected (figure 9). Several ships have experienced a sudden loss of directional control during their operation, a phenomenon known as *broaching*. They are more vulnerable when approached from the stern by steep and relatively long waves with celerity near to the ship's speed. Broaching is, before anything else, a yaw stability problem. However, in the course of the tight turn that follows the 'loss of heading', capsize may be incurred. For a study of this, 'post-critical', stage of broaching, the combined yaw–roll dynamics need to be considered. It seems that Davidson (1948) was the first to point out that a directionally stable vessel in calm water could become unstable in following waves. Wahab & Swaan (1964), Eda (1972) and Motora *et al.* (1981) provided further insights. On the basis of linearized equations, they showed that for the 'right wave', instability will occur when the ship lies on the down slope with the stern near the crest. Experiments of broaching on the basis of free-running models in square basins were pioneered by Nicholson (1974), Fuwa *et al*. (1982) and Marshfield (1987). The application of a nonlinear-dynamics approach recently produced a fundamental understanding about the phenomena underlying broaching. The key concepts and developments are summarized in § 4.

Surf-riding is another nonlinear type of behaviour, having its origin in the surge dynamics. It is particularly interesting because it can work as a precursor to broaching. A ship may be forced to advance with a speed equal to the wave celerity, despite the fact that the propeller thrust corresponds to a considerably lower, or higher, speed. Such behaviour was first studied by Grim (1951), and was reconsidered by him later (Grim 1983), pointing out the analogy with the behaviour of a pendulum under constant torque. Experimental evidence was provided by Du Kane & Goodrich (1962). Kan (1990) investigated the evolution of asymmetric surging and the transition to surf-riding when the peak of the fluctuating surge velocity reaches the wave celerity. Similar studies were undertaken in the same period by Umeda (1990). Surf-riding can trigger broaching when the angle between the direction of the waves and the ship is non-zero (quartering waves). In this laterally expanded space, the surf-riding states belong to a closed curve, of which fold and Hopf bifurcations are common features. Chaotic surf-riding based on the Feigenbaum scenario has been predicted (Spyrou 1995, 1996*b*).

For a craft moving at a very high speed, some part of the vertical lift force supporting the hull is of a hydrodynamic nature due to the occurrence of *planing*. High-speed craft in planing conditions are known to exhibit a variety of dynamic instabilities. A very well-known case is porpoising, an instability of the coupled heave and pitch (Martin 1978; Troesch & Falzarano 1992). Its simplest manifestation is a self-sustained oscillation in calm sea (implying the occurrence of a Hopf bifurcation event) when the speed exceeds a certain threshold. Other known types of unconventional behaviour are sudden heel, chine walking (roll oscillation), bow diving, and the pitch–yaw–roll oscillations known as 'corkscrew' (Blount & Codega 1992). Generally, high speed is believed to affect dynamic stability very considerably, and, with the current emphasis on high-speed maritime transportation, this area is expected to receive considerable attention in the near future.

5. The instabilities of the stern quartering sea

The work on ship stability of Paulling and co-workers at the University of California at Berkeley, although not falling into the domain of nonlinear dynamics, has inspired many developments in this field. The experiments at San Francisco Bay (Chow *et al*. 1974) consolidated the idea that a ship may capsize in a following sea environment when any of the three 'fundamental' instability mechanisms below occur.

(a) **Pure loss of stability.** Sudden, non-oscillatory capsize due to slow passage from a negative restoring region (around the wave crest).

(b) **Parametric instability.** Gradual build-up of rolling due to internal forcing coming from time-dependent (strictly speaking position-dependent) restoring.

(c) **Broaching.** The sudden loss of controllability in steep waves approaching from the stern. It gives rise to large heel, and, possibly, to capsize during the tight turn triggered by the loss of control.

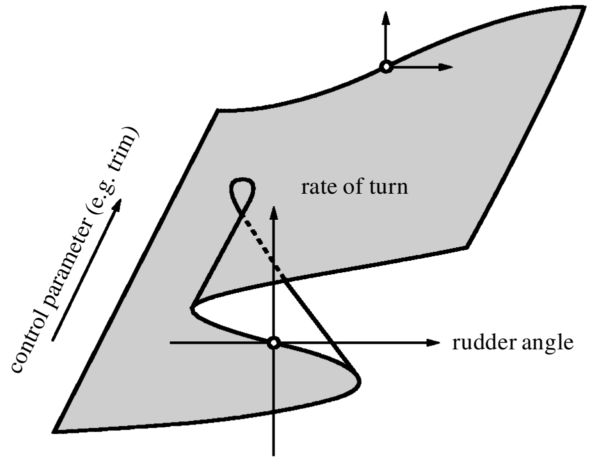

Figure 9. Steering diagram under the effect of a varying parameter such as trim.

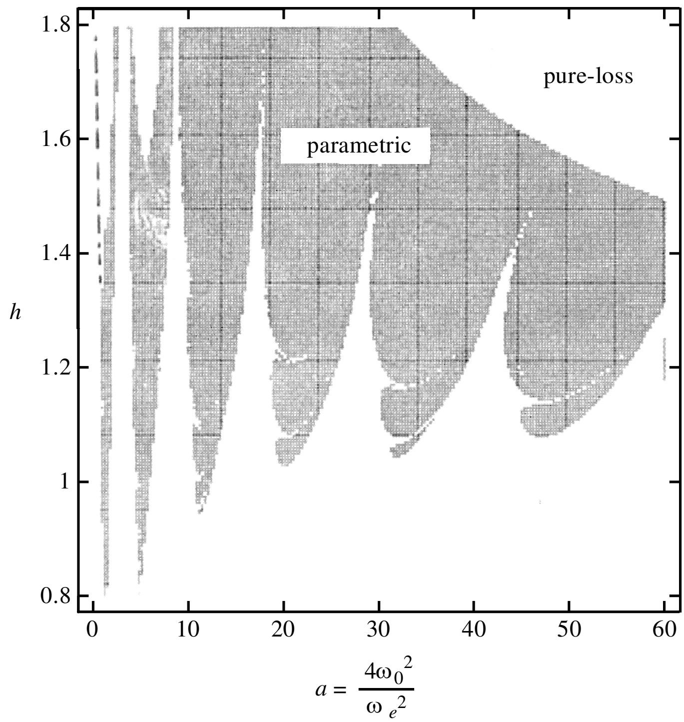

Figure 10. See opposite for description.

(a) *A unified approach for 'pure loss' and parametric instability*

The qualitative dynamics of pure loss and parametric instability can be represented by a nonlinear 'Mathieu' equation with some small damping. The characteristic of pure loss is that the instability is inflicted and the capsize occurs during a single passage from a crest. The three main factors that determine survivability are:

(a) whether the restoring becomes negative in some region around the wave crest;

(b) the magnitude of some initial heel and/or roll velocity as the ship enters into the negative restoring region;

(c) whether the speed of the ship relative to the wave celerity is sufficiently low that, given the negative restoring and the initial off-balance position, a heel angle finds the time to develop up to dangerous levels.

Pure loss is an intrinsically transient phenomenon. The parametric instability on the other hand is traditionally examined on the basis of steady-state stability analysis of the trivial upright equilibrium state using as parameters the frequency ratio and the wave steepness. In a practical context, however, what is important is whether capsize-angle levels are reached within a small number of wave-crest encounters. This calls for resetting the focus on the transient dynamics. When the considered number of waves is small, the required amplitude of variation of restoring can be considerably in excess of the corresponding steady-state amplitude, particularly for the principal and the fundamental resonance, which are the most relevant to ships. An approach based on transient dynamics allows for unification of the treatments of pure loss and parametric instability and for developing an assessment on the basis of a single diagram like figure 10 (Spyrou 2000).

Although Mathieu's equation probably 'captures' the fundamental dynamics, consideration of extra features—like water shipped on deck, change of damping due to deck submergence, etc.—would certainly add 'new dimensions' to this analysis. Another matter of concern is the nature of the excitation: ocean waves very rarely constitute a 'monochromatic sea'. Even in such a highly idealized environment, the geometry of real ships' hulls could only result in a cyclic variation of restoring by coincidence. Subsequently, a Hill-type equation would be deemed more appropriate, but there is little possibility that some specific form could be adopted as a generic model of design excitation. Not only is the excitation non-cyclic, but its main characteristics may be maintained only for limited time. For this reason, the asymptotic stability information derived from the Strutt diagram should be treated with caution. Viewing these problems from a different perspective, several authors have considered a stochastically varying parametric term (see, for example, Roberts 1982), aiming to develop some kind of 'statistical' ship-stability criterion.

Coupling with other motions will undoubtedly bring about a number of significant new effects into the dynamics: coupling of roll with lateral and rudder motions should influence the build-up of roll as the heading of the ship will be changing and a sway force will be excited. Another influence, examined in detail by Spyrou in this issue (pp. 1813–1834), arises when the ship cannot move with a nearly constant surge velocity due to the relative strength of the longitudinal wave force.

(*b*) *Recent insights into broaching*

Recent studies of broaching have revealed a rich content in terms of nonlinear dynamics (Spyrou 1996*a*, *c*, 1997*a*). Broaching may occur during the transition

Figure 10. A unifying diagram for capsize due to pure loss and due to parametric instability. The domain of capsize due to the parametric scenario is based on escape within eight cycles. The white regions that exist inside the principal and fundamental resonance regions indicate the occurrence of a 'quick' parametric-type capsize.

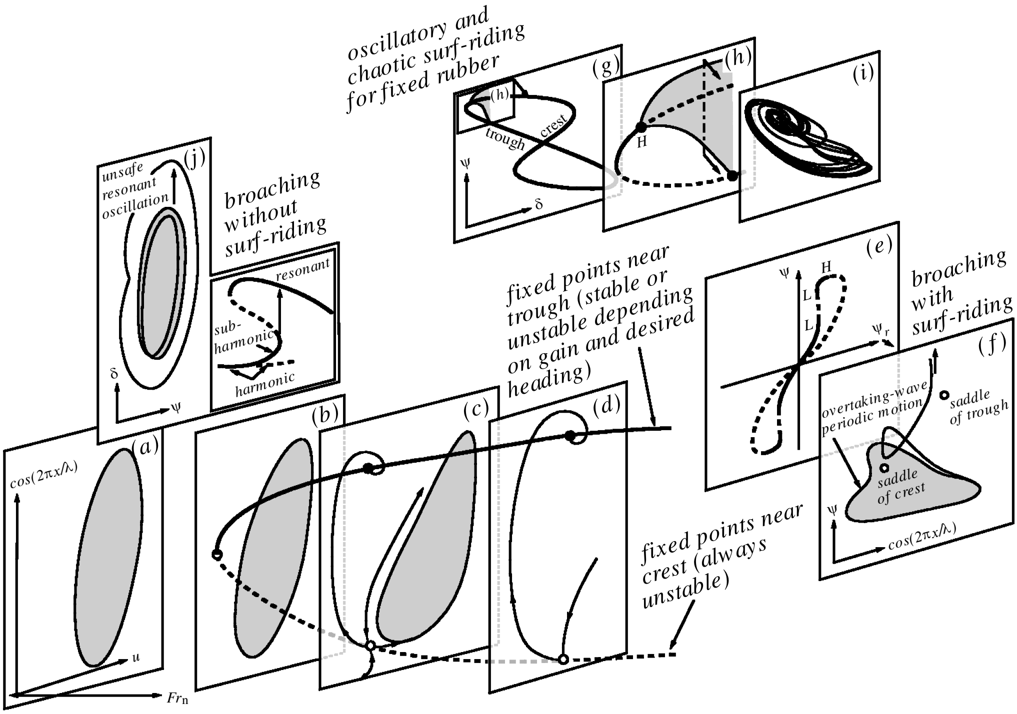

Figure 11. Overview of the nonlinear dynamics of broaching.

towards surf-riding, or it may occur at a lower speed (and usually with higher waves) due to loss of stability of the ordinary periodic yaw motion. The sequence of the phenomena involved in the two routes, under gradual increase of the nominal Froude number, is summarized in figure 11.

The evolution of the qualitative surge dynamics and the onset of surf-riding are shown on planes (a)–(d) in figure 11. When the heading of the ship does not coincide with the wave direction, the surf-riding equilibria belong to a closed curve having unstable (in surge) points near the crest and stable or unstable points (depending on the proportional gain of the autopilot) near the trough (plane (e)). When a ship operates with a non-zero heading relative to the wave, and the homoclinic connection event (see planes (c) and (d)) destroys the periodic response, the remaining possible motions are either surf-riding or a rapid turning motion that corresponds to broaching. Which of the two will occur depends on whether or not the autopilot gain is sufficient for attracting the orbit leaving the periodic pattern, and, thus, creating capture into stable surf-riding near the trough. An example of broaching is presented on plane (f). It was discovered that surf-riding could be realized in a periodic sense (Hopf bifurcation) and even in a chaotic sense (period-doubling cascade). On planes (g)–(i) the key phenomena can be seen along with their sequence when the angle of the rudder, δ, is treated as the control variable (Spyrou 1996b).

Finally, we describe the occurrence of broaching directly from the periodic pattern and without serious involvement of the surge dynamics (plane (j)). At a critical heading (for a certain ship and with fixed wave characteristics), a flip bifurcation occurs, creating a stable subharmonic response. This causes a rapid increase in the

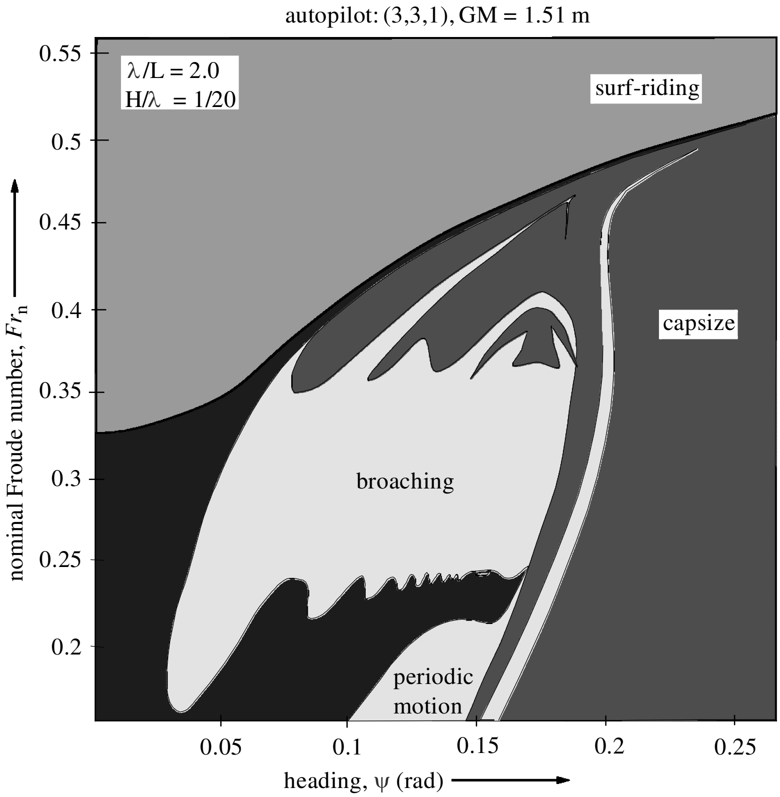

Figure 12. 'Multiple-effect' global analysis targeting the process of escape from surf-riding. The layout of the broaching domain and the domain of capsize eroding the broaching domain are shown (Spyrou 1997*b*).

amplitude of yaw oscillation, leading, shortly, to a backwards turn of the response curve, which precipitates a sudden and dangerous jump to resonance. The ensuing transient behaviour will correspond to cumulative-type broaching (Spyrou 1997*a*).

The transient dynamics of broaching have been studied thus far in respect of two specific transitions. The first transition referred to the process of escape from surf-riding after voluntary reduction of engine power or change of heading (see figure 12 and Spyrou (1996*c*)). A second scenario considered targeted the disappearance of the periodic motion due to the homoclinic connection bifurcation (Spyrou 1997*b*).

6. Future directions for research

The field of nonlinear ship dynamics is only now approaching maturity, and it is believed that several exciting new developments will be realized in the near future. In the authors' opinion, some of the areas in which further activity is expected, or would be very welcome, are the following.

(1) Integration of the probabilistic character of the seaway into the study of nonlinear dynamics.

(2) More rational description of hull loads taking better account of the detailed shape of a ship hull.

(3) Development of mathematical models of coupled motions.

(4) Instabilities that are intrinsic to high-speed operation.

(5) Development of improved techniques for analysing multi-degree dynamical systems.

(6) Use of the concepts of nonlinear dynamics in the modelling process.

(7) Making the link between nonlinear dynamics and design.

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